PERFORMANCE AND ROBUSTNESS OF FLOW CONTROLLERS DESIGNED USING NON-CAUSAL UNCERTAINTY BLOCKS

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ABSTRACT

According to some recent results, use of non-causal uncertainty blocks may be advantageous in the robust controller design of systems with multiple uncertain time-delays. In this work, performance and robustness improvements obtained by utilizing such an approach are presented. The flow controller design problem for networks with multiple uncertain time-delays is considered as a case study. It is shown that higher performance levels and larger stability margins are, in general, obtained by using non-causal uncertainty blocks. A number of simulations, which illustrate the time-domain performance improvement, are also presented.

Key Words: Robust control, $H_\infty$-optimization, time-delay systems

I. INTRODUCTION

Design of robust controllers for systems which involve uncertain time-delays has been a challenge [6]. Several approaches have been proposed in [13,5,12,4], among others, to solve the $H_\infty$-optimal controller design problem for systems with a single time-delay. The general solution to the $H_\infty$-optimal controller design problem for systems with multiple time-delays was then presented by Meinsma and Mirkin [3]. The approach of [3] requires taking the nominal time-delays outside the plant. The uncertain parts of the time-delays then appear in an uncertainty block. If the uncertain part of the time-delays are allowed to be negative, however, the uncertainty block becomes non-causal. In this case, a difficulty arises in the $H_\infty$-optimal controller design, since the small gain theorem [22,11], which is used in this approach, is not valid for non-causal systems in general (see example 1 in [18]). Recently, however, Ünal and Iftar [18] (also see [19] for an extension) showed that, under certain conditions, a small gain theorem is valid for systems with non-causal blocks.

A good example of a system which involves multiple uncertain time-delays is the data-communication network [7,23,2]. In order to avoid traffic congestion in a data-communication network, the flow controllers are designed in order to control the rate of data flow. An $H_\infty$-based flow controller design approach, which uses the design techniques of [13], was proposed in [8]. The implementation of this controller was later illustrated in [9]. In [8,9], however, the uncertain time-delays were assumed to be time-invariant and the controller was designed for the multiple time-delays considering the longest delay and equalizing the delays in the other channels to the longest one. The case of uncertain time-varying multiple time-delays was later considered in [10], where the controller was obtained by defining separate $H_\infty$ control problems for each channel. Therefore, the overall solution presented in [10] is not optimal, but suboptimal in the $H_\infty$ sense, in general. To find an optimal solution to this problem, the approach of [3] was first considered in [1]. Then, in [15,16], the same approach was used to obtain an $H_\infty$-optimal solution to the problem presented in [10]. However, in order to avoid non-causal uncertainty blocks, in [15,16], it was assumed that the uncertain parts of the time-delays are always non-negative. This assumption introduces two disadvantages: (i) the best performance is obtained, not for the plant with most probable time-delays, but for the plant with minimum time-delays; and (ii) robustness range must be larger, since the absolute value of the maximum allowable variation on the time-delays must now be as much as twice, compared to the case when the nominal time-delays are chosen as the average of the minimum and maximum possible time-delays. There exist two ways of overcoming this problem: (i) converting the problem to an equivalent problem without non-causalities using some manipulations (see remark 3 in [17]); and (ii) directly using non-causal uncertainty blocks, by utilizing the main result of [19]. As argued in [17], given the result of [19], it is more straightforward and natural to use the second approach. Hence, this latter approach was used in [17] to design $H_\infty$-optimal flow controllers.

In the present work, performance and robustness improvements obtained by utilizing the approach used in [17] are presented. The results of [18] and [19] are first summarized in Section II. The flow control problem is then introduced and the controller design approach of [17] is summarized in Section III. In Section IV, it is shown that higher performance levels and larger stability margins are in general obtained by using the approach of [17]. A number of simulations, which illustrate the time-domain performance improvement, are presented in Section V.
1.1 Notation

The notation is borrowed from [21,18]. \( \mathbb{R} \) denotes the set of real numbers, \((\cdot)^t\) denotes the transpose of \((\cdot)\), \(\text{diag} (\cdot)\) denotes a diagonal matrix with \((\cdot)\) on its diagonal, and \(\text{blkdiag} (\cdot)\) denotes a block diagonal matrix with blocks \((\cdot)\) on its diagonal. For a positive integer \( n \), \( \mathbb{R}^n \) denotes the \( n \)-dimensional real vector space and \( I_n \) denotes the \( 1 \times n \) dimensional matrix of all 1’s.

For each real \( p \in [1, \infty) \) and \( a \in \mathbb{R} \), \( L_p[a, \infty) \) denotes the space of all measurable functions \( f : [a, \infty) \to \mathbb{R} \) such that \( \int_a^\infty |f(t)|^p \, dt < \infty \). The norm on \( L_p[a, \infty) \) is defined as \( \|f\|_p := \left( \int_a^\infty |f(t)|^p \, dt \right)^{1/p} \), where \( a \in \mathbb{R} \) denotes the initial time. For each real \( p \in [1, \infty) \), positive integer \( n \), and \( a = [a_1 \ldots a_n] \in \mathbb{R}^n \), the set \( L_p^n[a, \infty) \) consists of all \( n \)-tuples \( f = [f_1 \cdots f_n] \), where each \( f_i \in L_p[a_i, \infty) \) for \( i = 1, \ldots, n \). The norm on \( L_p^n[a, \infty) \) is defined as \( \|f\|_p := \left( \sum_{i=1}^n \|f_i\|_p^p \right)^{1/p} \). Suppose \( f = [f_1 \cdots f_n] \), where \( f_i : [a_i, \infty) \to \mathbb{R} \) for \( i = 1, \ldots, n \). Then, for each finite \( T, f_T := [(f_1)_T \cdots (f_n)_T] \), called truncation of \( f \), where \( (f_i)_T : [a_i, \infty) \to \mathbb{R} \) for \( i = 1, \ldots, n \), is defined as

\[
(f_T)_t(t) := \begin{cases} 
0, & \forall t \geq a_i, if T < a_i \\
 f_i(t), & a_t \leq t \leq T \\
0, & t > T 
\end{cases}
\]

The set \( L_p^n[a, \infty) \), where \( a = [a_1 \cdots a_n] \), consists of all \( f = [f_1 \cdots f_n] \), where \( f_i : [a_i, \infty) \to \mathbb{R} \) for \( i = 1, \ldots, n \), with the property that \( f_T \in L_p^n[a, \infty) \) for all finite \( T \), and is called the extension of \( L_p^n[a, \infty) \) or the extended \( L_p^n[a, \infty) \)-space.

A mapping \( A : L_p^n[a, \infty) \to L_p^n[a, \infty) \), where \( a \in \mathbb{R}^n \) and \( a_2 \in \mathbb{R}^n \), is said to be \( L_p \)-stable with finite gain (\( L_p \)-sgf) if there exist non-negative finite constants \( \gamma \), called the gain of \( A \), and \( \beta \), called the bias of \( A \), such that, \( \|Ax\|_p \leq \gamma \|x\|_p + \beta \), for all \( x \in L_p^n[a, \infty) \). A mapping \( A : L_p^n[a, \infty) \to L_p^n[a, \infty) \) is said to be causal if \((Af)_T = (Af_T)_T \), for all finite \( T \), \( \forall f \in L_p^n[a, \infty) \).

II. SMALL GAIN THEOREMS

In this section, first we summarize the main result of [18]. Then, a more relaxed condition for the validity of the small-gain theorem under non-causal subsystems, given in [19], is presented. Consider the feedback configuration shown in Fig. 1, where \( u_1 \in L_p^n[a, \infty), e_1, y_1 \in L_p^n[a, \infty) \), \( u_2 \in L_p^n[a, \infty), e_2, y_2 \in L_p^n[a, \infty) \), and \( e_1, y_1 \in L_p^n[a, \infty) \). We assume that \( G_1 : L_p^n[a, \infty) \to L_p^n[a, \infty) \) and \( G_2 : L_p^n[a, \infty) \to L_p^n[a, \infty) \) are \( L_p \)-sgf, respectively with gain \( \gamma_1 \) and \( \gamma_2 \) and bias \( b_1 \) and \( b_2 \); i.e.,

\[
\|G_1 e_1\|_p \leq \gamma_1 \|e_1\|_p + b_1, \quad \forall e_1 \in L_p^n[a, \infty)
\]

(1)

and

\[
\|G_2 e_2\|_p \leq \gamma_2 \|e_2\|_p + b_2, \quad \forall e_2 \in L_p^n[a, \infty).
\]

(2)

When \( G_1 \) and \( G_2 \) satisfy (1) and (2) and are both causal, the small gain condition, \( \gamma_1 \gamma_2 < 1 \), proves the stability of the closed-loop system shown in Fig. 1 [21]. This result, however, does not directly extend to the case when at least one of the blocks is non-causal, as shown by an example in [18].

Now, let us assume that at least one of the blocks in Fig. 1 is non-causal, but the two cascade connections of these blocks (i.e., the two systems obtained by breaking the loop in Fig. 1(i) at \( e_1 \) (ii) at \( e_2 \)) are both causal; i.e., \( G_1 \) and \( G_2 \) satisfy

\[
(G_1 G_2 e_2)_T = ((G_1 G_2) (e_2))_T,
\]

(3)

\[
\forall e_2 \in L_p^n[a, \infty), \quad G_1 G_2 e_2 \in L_p^n[a, \infty), \quad \text{and}
\]

\[
(G_1 G_2 e_2)_T = ((G_1 G_2) (e_2))_T,
\]

(4)

\[
\forall e_2 \in L_p^n[a, \infty), \quad \text{for all } T. \text{ Also assume that } G_1 \text{ and } G_2 \text{ satisfy}
\]

\[
\|G_1(e_2 \pm e_1)\|_p \leq \|G_1(e_2)\|_p + \|G_1(e_1)\|_p.
\]

(5)

\[
\forall e_2, e_1 \in L_p^n[a, \infty), \quad \text{and}
\]

\[
\|G_2 (e_2 \pm e_1)\|_p \leq \|G_2 (e_2)\|_p + \|G_2 (e_1)\|_p.
\]

(6)

\[
\forall e_2, e_1 \in L_p^n[a, \infty), \quad \text{for all } T. \text{ Note that the class of systems which satisfy (5)–(6) is fairly large. In particular, these relations are satisfied by any linear } G_1 \text{ and } G_2.
\]

The main result of [18] can then be stated as follows:

**Theorem 1** [18]. Consider the feedback configuration shown in Fig. 1. Let \( G_1 \) and \( G_2 \) satisfy (1)–(6). Suppose \( \gamma_1 \gamma_2 < 1 \). Then the closed-loop system, i.e., the map from \( u = [u_1' \ u_2'] \) to \( y = [y'_1 \ y'_2]' \) (or to \( e = [e_1' \ e_2'] \)), is \( L_p \)-sgf.

In the flow control problem to be considered in the next section, a small gain theorem for the configuration shown in Fig. 1 is needed. In this case, \( n_1 = n_2 = 1 \), and both blocks are linear and stable. Furthermore, the causality assumption (3) is also satisfied. The other causality assumption, (4), however, is not necessarily satisfied. Instead, it has been recognized in [19], that a more relaxed condition is satisfied. Furthermore, it has been shown that this condition, given as (7) in the next theorem, is also sufficient for stability.

**Theorem 2** [19]. Consider the feedback configuration shown in Fig. 1, where \( u_1 \in L_p^n[a, \infty), \quad e_1, y_1 \in L_p^n[a, \infty), \quad u_2 \in L_p^n[a, \infty), \quad e_2, y_2 \in L_p^n[a, \infty), \quad \text{ where } a \in \mathbb{R}^n \) and \( \tau \in \mathbb{R} \). Let \( G_1 : L_p^n[a, \infty) \to L_p^n[a, \infty) \) and \( G_2 : L_p^n[a, \infty) \to L_p^n[a, \infty) \). It
IS assumed that both $G_1$ and $G_2$ are linear (thus (5) and (6) are satisfied) and $L_{psf}$ with gain $\gamma_1$ and $\gamma_2$ respectively and bias zero (thus (1) and (2) are satisfied with $b_1 = b_2 = 0$. For $i = 1, \ldots, n$, let $G_{ii}: L_{ps}([a_i, \infty]) \to L_{ps}([\bar{a}_i, \infty])$ denote the map from the $i^{th}$ input of $G_1$ to its output and $G_{ji}: L_{ps}([\bar{a}_i, \infty]) \to L_{ps}([a_j, \infty])$ denote the map from the input of $G_2$ to its $i^{th}$ output (i.e., $G_i = [G_{ii} \cdots G_{ij}]$) and $G_j = [G_{ji} \cdots G_{jj}]$). Suppose that $G_1G_2$ is causal (i.e., (3) is satisfied). Also suppose that $G_2G_{ii}$ is causal, i.e.,

$$G_{ii} = \left((G_{ii})(e_i)\right)_T, \quad \forall e_i \in L_{p}(\{a_i, \infty\}), \quad \forall T, \quad \text{for all } i = 1, \ldots, n. \quad \text{(7)}$$

In this section, a flow control problem is presented and its solution, as proposed in [17], is summarized. A data-communication network with $n$ sources feeding a single bottleneck node is considered. The flow controller is designed to regulate the queue length at the bottleneck node, despite all time-varying uncertainties in the time-delays. The controller, which has to be implemented at the bottleneck node, calculates a rate command for each source to adjust the rate of data that is sent to the bottleneck node. The dynamics of the queue length are given as [10]:

$$\dot{q}(t) = \sum_{i=1}^{n} r_i(t) - c(t) \quad \text{(8)}$$

where,

$q(t)$ is the queue length at the bottleneck node at time $t$,

$r_i(t)$ is the rate of data received by the bottleneck node at time $t$ from the $i^{th}$ source, $i = 1, \ldots, n$,

$c(t)$ is the outgoing rate of data from the bottleneck at time $t$, which equals to the capacity of the outgoing link assuming that $q(t)$ is positive.

The rate of data received by the bottleneck node at time $t$ is given as follows [10]:

$$r_i(t) = \begin{cases} (1 - \delta_0)(t) q(t - \tau_i(t)), & t - \tau_i'(t) \geq 0 \\ 0, & t - \tau_i'(t) < 0, \end{cases} \quad \text{(9)}$$

where

$r_i(t)$ is the rate command for the $i^{th}$ source issued by the controller at time $t$.

Here, $\tau_i(t) = \tau_i^0(t) + \tau_i'(t)$ is the total round-trip time-delay in the $i^{th}$ channel, where

$\tau_i^0(t) = h_i + \delta_i^0(t)$ is the backward time-delay at time $t$, which is the time required for the data sent from the $i^{th}$ source to reach the bottleneck node. Here, $h_i$ is the time-invariant known backward nominal time-delay, and $\delta_i^0(t)$ is the time-varying backward time-delay uncertainty.

$\tau_i'(t) = h_i' + \delta_i'(t)$ is the forward time-delay at time $t$, which is the time required for the data sent from the $i^{th}$ source to reach the bottleneck node. Here, $h_i'$ is the time-invariant known forward nominal time-delay, and $\delta_i'(t)$ is the time-varying forward time-delay uncertainty.

The total round-trip time-delay in the $i^{th}$ channel can also be expressed as $\tau_i(t) = h_i + \delta_i(t)$, where $h_i = h_i^0 + h_i'$ is the nominal round-trip time-delay and $\delta_i(t) = \delta_i^0(t) + \delta_i'(t)$ is the time-varying round-trip time-delay uncertainty in the $i^{th}$ channel of the system. Although the actual problem involves some hard constraints (non-negativity and capacity constraints on the queue length and flow rates), for the purpose of controller design, these constraints are assumed to be satisfied at all times. It has been verified by a vast number of simulations that controllers designed under this assumption also work well under real conditions when these constraints are active [10,1,15,16,19,17].

Defining

$$q_0(t) := \int \left[ \sum_{i=1}^{n} r_i(t - h_i) - c(t) \right] dv + q(0)$$

as the nominal queue length, from (8) and (9), $q(t)$ is given as $q(t) = q_0(t) + \delta(t)$, where $\delta(t)$ is the uncertainty in the queue length. Proceeding as in [1,16,17] we obtain $\delta(t) = \sum_{i=1}^{n} \delta_i(t)$, where $\delta_i(t)$ is the output of the system shown in Fig. 2. Here, $r_i(t) := r_i(t - h_i)$, $M_i$ represents multiplication by $\phi_i$, and $\varphi_i, \varphi_i'$ are constants to be specified later.

Note that, the linear time-varying blocks, $\Delta_1$ and $\Delta_2$ in Fig. 2, are non-causal if $\delta(t)$ is negative. To overcome this difficulty, in [15,16], it was assumed that

$$0 \leq \delta(t) < \delta^* \quad \text{(10)}$$

for some positive bound $\delta^*$. This assumption, however, requires taking $h_i$’s as the minimum possible time-delays, rather than as the nominal (i.e., most probable) time-delays. This, in turn, causes optimization of the performance not for the actual nominal plant, but for the plant with minimum possible time-delays. Furthermore, this also requires taking

$\tau_0(t) = h_1 + \delta_1(t)$ is the forward time-delay at time $t$, which is the time required for the data sent from the $i^{th}$ source to reach the bottleneck node. Here, $h_i'$ is the time-invariant known forward nominal time-delay, and $\delta_i'(t)$ is the time-varying forward time-delay uncertainty.
the bounds $\delta_i^*$ larger (as much as twice), which in turn introduces conservativeness in the robust controller design.

In [17], using Theorem 2, the system in Fig. 2 is allowed to be non-causal and, instead of (10), it is assumed that

$$|\delta_i(t)| < \delta_i^*$$  \hspace{1cm} (11)

for some positive bound $\delta_i^*$, with the additional condition that

$$\tau_r(t) = h_i + \delta_i(t) \geq 0, \forall t.$$  \hspace{1cm} (12)

In this way, $h_i$'s can be chosen as the most probable time-delays and $\delta_i^*$ can be chosen smaller than the bound in (10).

As in [10,1,15,16,17], in addition to the bound in (11), it is also assumed that the time derivatives of the total and the forward time-delay uncertainties are bounded as $|\dot{\delta}_i(t)| < \beta_i$ and $|\dot{\beta}_i(t)| < \beta_i'$, respectively, for some $0 < \beta_i' \leq \beta_i < 1$. We note that the bounds $\delta_i^*$, $\beta_i$, and $\beta_i'$ are design parameters. Their effects on the performance level and robustness are analyzed in [20].

It can be shown that (see [10,16]) the $L_2$-induced norm of each delay block in $\Delta_{i1}$, shown in Fig. 2, is less than

$$\frac{1}{\sqrt{1 - \beta_i^2}}.$$  \hspace{1cm} (13)

On noting that the $L_2$-induced norm of the multiplication blocks are bounded by the bounds of the multipliers, the $L_2$-induced norm of $\Delta_{i1}$ is less than $\frac{\beta_i + \beta_i'}{\sqrt{1 - \beta_i^2}}$. Thus, choosing $\phi_{i1} = \sqrt{\frac{\beta_i + \beta_i'}{1 - \beta_i^2}}$, the $L_2$-induced norm of $\Delta_{i1}$ becomes less than $\frac{1}{\sqrt{2}}$.

To find a bound on the $L_2$-induced norm of $\Delta_{i2}$, as in [19], let us define $\hat{u}_i(t) := \int_{t-\delta_i}^{t} r_h^k(v)dv$ and $\hat{v}_i(t) := \int_{t-\delta_i}^{t} r_h^k(v)dv$. Note that, both $\|\hat{u}_i\|_2$ and $\|\hat{v}_i\|_2$ are less than or equal to $\delta_i^* \|r_h^k\|_2$, with equality holding for some $r_h^k$. Assuming that $\delta_i(t)$ is not identically zero (in which case the $L_2$-induced norm of $\Delta_{i2}$ would be zero), let $t_1, t_2, \ldots$ indicate the times when $\delta_i(t)$ change sign. Note that, by the assumption that $|\delta_i(t)|$ is bounded, there are countably many such points and $\delta_i(t)$ is equal to zero at these points. Without loss of generality, let $0 = t_0 < t_1 < t_2 < \ldots$. Then, the square of the $L_2$-induced norm of the integral block in $\Delta_{i2}$ can be written as

$$\sup_{1 \leq a \leq L} \left\{ \sum_{t_{a-1}}^{t_a} \left[ \int_{t_{a-1} - \delta_i(t)}^{t_a} r_h^k(v)dv \right]^2 dt \right\}.$$

Thus, any $r_h^k$ which achieves the norm must be sign-invariant in any interval $(t_{a-1}, t_a)$. Now suppose that, at a given $t$, $\delta_i(t) \equiv 0$. Then, for any $r_h^k$, which is sign-invariant, we have $\int_{t_{a-1} - \delta_i(t)}^{t_a} r_h^k(v)dv \leq |\hat{v}_i(t)|$. On the other hand, if $\delta_i(t) < 0$, then for any $r_h^k$, which is sign-invariant, we have $\int_{t_{a-1} - \delta_i(t)}^{t_a} r_h^k(v)dv \geq |\hat{v}_i(t)|$. The above analysis show that the $L_2$-induced norm of $\Delta_{i2}$ is less than or equal to $\frac{\delta_i^*}{\phi_{i2}}$. Thus, choosing $\phi_{i2} = \sqrt{\frac{\beta_i + \beta_i'}{1 - \beta_i^2}}$, the $L_2$-induced norm of $\Delta_{i2}$ becomes less than or equal to $\frac{1}{\sqrt{\beta_i}}$. Then, the $L_2$-induced norm of $\Delta_i := \left[ \begin{array}{c} \Delta_{i1} \\ \Delta_{i2} \end{array} \right]$ is less than 1.

The following performance requirements must also be considered in addition to robust stability [10]:

- **Tracking requirement:** $\lim_{t \to \infty} q(t) = q_d$, where $q_d > 0$ is the desired queue length.
- **Weighted fairness requirement:** $\lim_{t \to \infty} c(t) = c_m$, where $\alpha > 0, i = 1, \ldots, n$, are the fairness weights, which satisfy $\sum_{i=1}^{n} \alpha_i = 1$, under the assumption that $\lim_{t \to \infty} c(t) = c_m$ exists.

In [15–17], to solve the above stated problem (i.e., to design a controller which robustly stabilizes the overall system and satisfies the performance requirements), a mixed sensitivity minimization problem, first considered in [1], for the system shown in Fig. 3 was defined. Here, $P_i(s) = \frac{1}{s}$ is the nominal plant, $K$ is the controller to be designed, $\Delta_i(s) = \text{diag}(e^{-h_1}, \ldots, e^{-h_n})$ represents the nominal time-delays which are taken outside the plant in order to apply the approach of [3], and $\Delta = \text{blkdiag}(\Delta_1, \ldots, \Delta_n)$ represents the uncertainties in the system and has $L_2$-induced norm less than 1. Furthermore, $W_i(s) = [W_{1i}(s) \cdots W_{ni}(s)]$, where $W_i(s) = \left[ \begin{array}{c} \phi_{i1} \\ \phi_{i2} \end{array} \right]$. Then, $W_i(s) = \frac{1}{s} W_i(s) = \sigma_i$, and

$$W_i(s) = \frac{1}{s} \left[ \begin{array}{ccc} \alpha_1 & -1 & 0 \\ \alpha_i & 0 & -1 \\ \vdots & \vdots & \vdots \\ \alpha_n & 0 & 0 \end{array} \right].$$
where \( \sigma_1 > 0 \) and \( \sigma_2 > 0 \) are design parameters, are weighting matrices. Moreover, \( d := q_i - c \), \( e_i \) is the weighted (by \( \sigma_1 \)) integral of the error \( y := q_i - q \) and is introduced to achieve fairness, and \( e_2 \) is introduced to achieve the weighted fairness requirement. The problem is then to design a controller \( K \), which stabilizes the closed-loop nominal system (the system shown in Fig. 3 with \( \Delta = 0 \)) and minimizes the \( H_{\infty} \) norm of the closed-loop transfer function matrix from \( w = \begin{bmatrix} w'_1 \end{bmatrix} \) to \( z = \begin{bmatrix} z'_1 \end{bmatrix} \).

In [15,16], this problem was solved assuming that the uncertain parts of the time-delays are bounded as in (10) and hence \( \Delta \) is causal. However, in [17], using Theorem 2, it is shown that the same design procedure can be applied if (11), with the additional condition (12), is used instead of (10). For this, for some \( \varepsilon > 0 \), let us define \( \tilde{W}_i(s) := \frac{s}{s + \varepsilon} W_i(s) \). Then, as it was shown in [19], by suppressing external signals (which do not have any effect on closed-loop stability), the system shown in Fig. 3 can be represented as in Fig. 1, where \( G_i = \tilde{W}_i \Delta_i \) and \( G_2 = \Delta_2 T \), where \( T := K (1 + P_\Delta K)^{-1} S + \varepsilon \) is the transfer function matrix from \( \tilde{w}_i = \tilde{W}_i \tilde{w}_i \) to \( r \). Note that, since \( \tilde{W}_i \) and \( \Delta_i \) are stable, (1) is satisfied. Furthermore, since \( K \) is chosen to stabilize the closed-loop nominal system (system shown in Fig. 3 with \( \Delta = 0 \)), the transfer function \( \tilde{w}_i \) to \( r \) is stable. Thus, in particular, \(-K (1 + P_\Delta K)^{-1} \Delta_i T \), which is the transfer function from the first entry of \( \tilde{w}_i \) to \( r \) and is strictly proper, is stable, (the pole at zero is canceled by the pole of \( P_\Delta \)). This implies that \( T \) is also stable. Therefore, since \( \Delta_i \) is stable, \( G_i \) is also stable, and hence (2) is also satisfied. (5) and (6) are satisfied since \( G_1 \) and \( G_2 \) are both linear.

To show that (3) is satisfied, note that \( G_1 G_2 = \tilde{W}_1 \Delta_1 \Delta_2 T \), where \( \Delta_1 = \text{blkdiag}(\Delta_1, \ldots, \Delta_n) \), where \( \lambda_i(s) := e^{-\lambda_i s} \) is the \( i \)-th diagonal element of \( \Delta_1(s) \). Also note that (see Fig. 2), the maximum time-advance in \( \Delta_i \) is \( \max_{i=1}^n (\Delta_i(t)) \), which is not greater than \( h_i \), by (12). Thus since \( \lambda_i \) is a pure delay of \( h_i \), for \( i = 1, \ldots, n \), each element of \( \Delta_1 \), hence \( \Delta_1 \) itself, is causal. Since \( \tilde{W}_i \) and \( T \) are also causal, this implies that \( G_1 G_2 \) is causal, and hence (3) is satisfied.

Finally, to show that (7) is satisfied, note that \( G_i = \tilde{W}_i \Delta_i \), where \( \tilde{W}_i(s) := \frac{\phi_{i-1}}{s + \varepsilon} \), and \( \phi_{i-1} := \frac{s \phi_i}{s + \varepsilon} \). As argued above, the maximum time-advance in \( \Delta_i \), and \( \Delta_2 \) is not greater than \( h_i \). Since \( \tilde{W}_i \) and \( \tilde{W}_2 \) are causal, this implies that the maximum time-advance in \( G_i \) is not greater than \( h_i \). On the other hand, \( G_2 = \tilde{W}_2 \Delta_2 T \), of which is causal. Therefore, since \( \lambda_i \) is a pure delay of \( h_i \), \( G_2 G_i \) is causal for all \( i = 1, \ldots, n \), and hence (7) is satisfied. Therefore, by Theorem 2, the small gain theorem can be applied to our system.

As shown in [19], the above result implies that we can proceed as in [15,16], by using (11) instead of (10), to design a robust stabilizing controller to solve the aforementioned sensitivity minimization problem. Utilizing the result in [19], the robustly stabilizing flow controller for data-communication networks was designed in [17] by using (11).

The details of the controller design approach can be found in [17]. The controller is linear and time-invariant. It is also optimal in the sense that the \( H_{\infty} \) norm of the closed-loop transfer function matrix from \( w = \begin{bmatrix} w'_1 \end{bmatrix} \) to \( z = \begin{bmatrix} z'_1 \end{bmatrix} \) in Fig. 3 is minimized. The controller structure is shown in Figure 8 of [17]. As indicated there, the controller involves a number of finite-impulse-response blocks, a proportional-integral block, and a block with a state-space dimension \( n + 1 \), where \( n \) is the number of sources feeding the bottleneck node. It should be noted that the controller structure and complexity is the same whether a causal or a non-causal uncertainty block is used. The computational burden to design the controller is also independent of which approach is used.

### IV. PERFORMANCE LEVELS AND STABILITY MARGINS

In this section, we will compare the performance levels and stability margins of the controllers designed by the approach of [15,16] to the controllers designed by the approach of [17] (i.e., by using (11) instead of (10) and allowing non-causal uncertainty blocks). For brevity, throughout this and the next section, the controller design using the approach proposed in [15,16] will be called the causal approach and the controller design using the approach proposed in [17] will be called the non-causal approach.

In both approaches, the designed controller, \( K^{\text{opt}} \), internally stabilizes the system and satisfies \( \|P_{\infty} \| \leq \gamma_{\text{opt}} := \inf_{K_{\text{stabilizing}}} \|T_{\infty} \| \) (for a different \( \gamma_{\text{opt}} \) for each approach), where

\[
T_{\infty} = \begin{bmatrix}
-\Lambda_i KSW_i & \Lambda_i KSW_2 \\
-WSW_i & WSW_2 \\
-W_i \Lambda_i KSW_i & W_i \Lambda_i KSW_2
\end{bmatrix}
\]

is the closed-loop transfer function matrix from \( w = \begin{bmatrix} w'_1 \end{bmatrix} \) to \( z = \begin{bmatrix} z'_1 \end{bmatrix} \) in Fig. 3, where \( S := (1 + P_\Delta K)^{-1} \). Since the optimal controller (besides stabilization) is aimed at minimizing the sensitivity level, \( \gamma_{\text{opt}} \), we will define its reciprocal, \( \frac{1}{\gamma_{\text{opt}}} \), as the performance level of the designed controller \( K^{\text{opt}} \).

To compare the performance levels and the stability margins of the controllers designed by the two approaches, we consider an example network with two sources. We consider four different cases for the possible time-delays in each channel. Further cases, which also support all the conclusions drawn below, can be found in [14]. The nominal time-delays and the uncertainty bounds for each channel in each case are given in Table I. In each case, the actual time-delay in channel \( i \) is assumed to vary from \( \tau_{\text{min}} - \delta_{\min} \) to \( \tau_{\text{max}} + \delta_{\max} \). Therefore, for \( i = 1, 2 \), for the causal approach we take

\[
h_i = \tau_{\text{min}} - \delta_{\max} \quad \text{and} \quad \delta_i = 2\delta_{\max}
\]

and for the non-causal approach we take...
\[ h_i = t_i^{\text{nom}} \quad \text{and} \quad \delta_i^* = \delta_i^{\text{max}}. \tag{14} \]

The other controller design parameters are given in Table II.

The performance level of the controller designed by the causal and the non-causal approach are shown in the second column of Tables III and IV, respectively, for each case. It is seen that the performance level of the controllers designed by the non-causal approach is greater than the performance level of the controllers designed by the causal approach in each case (by more than 10% in most cases and as much as 24% in case 4). Therefore, controllers designed by the non-causal approach are expected to perform better in terms of robustness and achieving tracking and weighted fairness requirements despite changes in the outgoing flow rate.

To define the actual stability margins on the time-delays and their derivatives, note that the system shown in Fig. 3 is robustly stable as long as \[ \| \Delta \| < \frac{1}{\rho} \] (here \( \| \cdot \| \) denotes the \( L_2 \)-induced norm), where \( \rho := \| T_{z_1 z_2} \| \), where \( T_{z_1 z_2} := -\Lambda KSW_1 \) is the closed-loop transfer function matrix from \( w_1 \) to \( z_1 \) in Fig. 3. Note that, with \( K = K^{\text{opt}}, \rho \leq \gamma^{\text{opt}}, \) since \( T_{z_1 z_2} := -\Lambda KSW_1 \) is a sub-block of \( T_{z_2} \). Note that \( \| \Delta \| < 1 \) when the stability margins on \( \delta(t), \dot{\delta}(t) \), and \( \dot{\delta}'(t) \) are respectively \( \delta_i^*, \beta_i^*, \) and \( \beta_i'^* \). Therefore, \( \| \Delta \| < \frac{1}{\rho} \) is satisfied if the actual stability margins on \( \delta(t), \dot{\delta}(t) \), and \( \dot{\delta}'(t) \) are respectively changed to \( \delta_i^{\text{act}}, \beta_i^{\text{act}} \) and \( \beta_i'^{*\text{act}} \), where \( \delta_i^{\text{act}} = \frac{1}{\rho} \delta_i^* \) and

\[ \frac{\beta_i^{\text{act}} + \beta_i'^{*\text{act}}}{\sqrt{1 - \beta_i^{\text{act}}}^2} = \frac{\beta_i^* + \beta_i'^*}{\rho \sqrt{1 - \beta_i^*}^2} \tag{15} \]

are satisfied for \( i = 1, \ldots, n \). Note that there are infinitely many solution for \( \beta_i^{\text{act}} \) and \( \beta_i'^{*\text{act}} \) in (15) (the system is robustly stable for any one of these solutions). To obtain unique solutions we introduce the additional constraint \( \frac{\beta_i^{\text{act}}}{\beta_i'^{*\text{act}}} = \frac{\beta_i}{\beta_i'^*} \).

The actual stability margins, together with the value of \( \rho \), are given in Table III and IV, respectively for the controller designed by the causal approach and the non-causal approach, in each case considered above (we list \( 2\delta_i^{\text{act}}, \) which is the length of the full stability range, rather than \( \delta_i^{\text{act}} \), in case of non-causal approach, so that we compare it to the length of the full stability range in the case of causal approach, which is simply \( \delta_i^{\text{act}} \)). It is seen that, in all the cases, the value of \( \rho \) is smaller, and as a result, all of the actual stability margins are greater under the non-causal approach, compared to the causal approach. This means that the controller obtained by the non-causal approach is more robust than the controller obtained by the causal approach against changes in the time-delays and their derivatives. Also note that the guaranteed stability range for the variations in the time-delays is centered around the nominal time-delays for the non-causal approach, i.e., the controller designed by the non-causal approach guarantees stability in the range \( t_i^{\text{nom}} - \delta_i^{\text{act}} < t_i(t) < t_i^{\text{nom}} + \delta_i^{\text{act}}, \) as long as \( |\dot{t}_i(t)| < \beta_i^{\text{act}} \) and \( |\dot{t}_i'(t)| < \beta_i'^{*\text{act}} \) are also satisfied. On the other hand, assuming \( |\dot{t}_i(t)| < \beta_i^{\text{act}} \) and \( |\dot{t}_i'(t)| < \beta_i'^{*\text{act}} \), the controller designed by the causal approach guarantees

### Table I. Nominal Time-Delays and Uncertainty Bounds.

<table>
<thead>
<tr>
<th>Case</th>
<th>( t_i^{\text{nom}} )</th>
<th>( \delta_i^{\text{max}} )</th>
<th>( \delta_i'^{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Case 2</td>
<td>1</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Case 3</td>
<td>1</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Case 4</td>
<td>4</td>
<td>2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### Table II. Controller Design Parameters for all cases.

<table>
<thead>
<tr>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_1' )</th>
<th>( \beta_2' )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.1</td>
<td>0.01</td>
<td>0.01</td>
<td>2/3</td>
<td>1/3</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### Table III. Performance Level and Stability Margins for the Causal Approach.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \frac{1}{\gamma^{\text{opt}}} )</th>
<th>( \rho )</th>
<th>( \delta_i^{\text{act}} )</th>
<th>( \delta_i'^{\text{act}} )</th>
<th>( \beta_i^{\text{act}} )</th>
<th>( \beta_i'^{*\text{act}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.6667</td>
<td>1.4986</td>
<td>0.1335</td>
<td>0.1335</td>
<td>0.1385</td>
<td>0.0679</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.3940</td>
<td>2.5354</td>
<td>0.5522</td>
<td>0.5522</td>
<td>0.0844</td>
<td>0.0407</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.3683</td>
<td>2.7137</td>
<td>0.3685</td>
<td>0.1843</td>
<td>0.0791</td>
<td>0.0381</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.1545</td>
<td>6.4606</td>
<td>0.6191</td>
<td>0.1548</td>
<td>0.0340</td>
<td>0.0162</td>
</tr>
</tbody>
</table>

### Table IV. Performance Level and Stability Margins for the Non-Causal Approach.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \frac{1}{\gamma^{\text{opt}}} )</th>
<th>( \rho )</th>
<th>( 2\delta_i^{\text{act}} )</th>
<th>( 2\delta_i'^{\text{act}} )</th>
<th>( \beta_i^{\text{act}} )</th>
<th>( \beta_i'^{*\text{act}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.6959</td>
<td>1.4316</td>
<td>0.1396</td>
<td>0.1396</td>
<td>0.1445</td>
<td>0.0710</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.4193</td>
<td>2.3789</td>
<td>0.5886</td>
<td>0.5886</td>
<td>0.0897</td>
<td>0.0433</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.4090</td>
<td>2.4372</td>
<td>0.4104</td>
<td>0.2052</td>
<td>0.0876</td>
<td>0.0423</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.1910</td>
<td>5.2321</td>
<td>0.7646</td>
<td>0.1912</td>
<td>0.0418</td>
<td>0.0199</td>
</tr>
</tbody>
</table>
Table V. Time-Delay Range for Guaranteed Stability.

<table>
<thead>
<tr>
<th>Channel 1</th>
<th>Causal</th>
<th>Non-Causal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>(0.9, 1.0335)</td>
<td>(0.9302, 1.0698)</td>
</tr>
<tr>
<td>Case 2</td>
<td>(0.3, 0.8522)</td>
<td>(0.7057, 1.2943)</td>
</tr>
<tr>
<td>Case 3</td>
<td>(1.5, 1.8685)</td>
<td>(1.7948, 2.2052)</td>
</tr>
<tr>
<td>Case 4</td>
<td>(2, 2.6191)</td>
<td>(3.6177, 4.3823)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel 2</th>
<th>Causal</th>
<th>Non-Causal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>(0.9, 1.0335)</td>
<td>(0.9302, 1.0698)</td>
</tr>
<tr>
<td>Case 2</td>
<td>(0.3, 0.8522)</td>
<td>(0.7057, 1.2943)</td>
</tr>
<tr>
<td>Case 3</td>
<td>(0.75, 0.9343)</td>
<td>(0.8974, 1.1026)</td>
</tr>
<tr>
<td>Case 4</td>
<td>(0.5, 0.6548)</td>
<td>(0.9044, 1.0956)</td>
</tr>
</tbody>
</table>

Table VI. Actual time-delays.

<table>
<thead>
<tr>
<th>Case</th>
<th>i</th>
<th>(\tau_i^{\text{nom}}(t))</th>
<th>(\tau_i^*(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>1</td>
<td>0.49 + 0.03 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.49 + 0.01 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.50 + 0.02 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.50 + 0.01 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td>2a</td>
<td>1</td>
<td>0.5 + 0.15 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.5 + 0.1 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.6 + 0.1 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.4 + 0.1 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td>3a</td>
<td>1</td>
<td>1.1 + 0.1 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.9 + 0.1 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.4 + 0.06 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.6 + 0.04 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td>3b</td>
<td>1</td>
<td>1.0 + 0.08 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.7 + 0.02 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.4 + 0.06 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.4 + 0.04 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td>3c</td>
<td>1</td>
<td>1.25 + 0.10 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.9 + 0.1 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.75 + 0.1 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.9 + 0.01 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td>4a</td>
<td>1</td>
<td>1.8 + 0.05 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>1.2 + 0.15 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.6 + 0.07 \sin \left( \frac{2\pi}{30} t \right)</td>
<td>0.4 + 0.03 \sin \left( \frac{2\pi}{80} t \right)</td>
</tr>
</tbody>
</table>

stability in the range \(\tau_i^{\text{nom}} - \delta_i^{\text{max}} < \tau_i(t) < \tau_i^{\text{nom}} - \delta_i^{\text{act}}\). This causes the guaranteed stability range to be centered below \(\tau_i^{\text{nom}}\) when \(\rho > 1\) and above \(\tau_i^{\text{nom}}\) when \(\rho < 1\). The guaranteed stability ranges for the four cases considered above are shown in Table V for both approaches. It is seen that for many cases the guaranteed stability range under the causal approach does not even include the nominal time-delay.

To obtain a further comparison (presumably a fairer one, as suggested by one of reviewers) we also undertake an alternative design approach. We first obtain the controller using the non-causal approach. Then take the lower bounds on the guaranteed stability ranges for each channel as \(\bar{h}_i\) and design a new controller using the causal approach. Then the lower bounds on the guaranteed stability ranges for each channel are naturally the same for the two approaches. Thus, the approach which provides a higher upper bound would be superior. By taking this approach, the upper bounds on the stability margins for the causal approach in channel 1 for cases 1 through 4 are respectively obtained as 0.9792, 0.8618, 1.8901, and 3.7350. For channel 2, we obtain 0.9792, 0.8618, 0.9451, and 0.9337. By comparing these values to the upper bounds obtained by the non-causal approach (as shown in Table V), it is seen that using the non-causal approach higher upper bounds (by more than 10% in all cases and as much as 50% in case 2) are obtained in all channels and in all cases.
V. TIME-DOMAIN PERFORMANCE

In this section, we will compare the time-domain performance of the controllers designed under causal approach to the time-domain performance of the controllers designed under non-causal approach using a number of simulations. We consider the same example network considered in the previous section and the same cases for the nominal time-delays and uncertainty bounds given in Table I. The controller design parameters $h_i$ and $\delta_i^+$ are calculated as given in (13) and (14), for the causal and non-causal approaches respectively. The other controller design parameters are given in Table II.

The simulations are performed using MATLAB Simulink, where non-linear effects (i.e., non-negativity and capacity constraints on the queue length and flow rates) are also taken into account. The buffer size (maximum queue length) is taken as 60 packets, the desired queue length, $q_d$, is taken as 30 packets, the capacity of the outgoing link (which equals to $c(t)$ when $q(t) > 0$) is taken as 90 packets/tu (where tu stands for time unit), and the rate limits of the sources are taken as 150 packets/tu in all cases. We consider a total of six different cases, where the actual time-delays are shown in Table VI. In this table, Case $k_a$, Case $k_b$, etc. refer to a case where the controller designed for Case $k$ of Table I is used ($k = 1, \ldots, 4$).

The simulation results are shown in Figs 4–9. In all figures, part (a) shows the results obtained for the controller designed using the causal approach and part (b) shows the results obtained for the controller designed using the non-causal approach. In all graphs, $q$ is the queue length, $q(t)$ (whose scale is shown on the right-hand-side of each graph), and $r_i'$ is the actual rate at which data is send from source $i$, $i = 1, 2$, (whose scale is shown on the left-hand-side of each graph).

Case 1a. In this case, the actual time-delays vary within the guaranteed stability range of both controllers, designed using the causal and the non-causal approach. Both controllers stabilize the system and produce a fast response as shown in Fig. 4. The tracking and weighted fairness
requirements are also satisfied, apart from small steady-state oscillations (not noticeable in the graphs), which are due to oscillations in the forward time-delays.

**Case 2a.** In this case, different from Case 1a, controllers are designed for a large variation in the time-delays. The actual time-delays vary within the guaranteed stability range of non-causal approach, however, not exactly within the guaranteed stability range of the causal approach. However, as shown in Fig. 5, not only the controller designed by the non-causal approach, but also, the controller designed by the causal approach stabilizes the system. Fig. 5 shows that both of the controllers achieve tracking and fairness requirements, however, the controller designed by the non-causal approach produces a faster response with no overshoot, compared to the response of the controller designed by the causal approach.

**Case 3a.** In this case, the actual time-delays vary within the guaranteed stability range of the controller designed by the non-causal approach, but outside the guaranteed stability range of the controller designed by the causal approach. As a result, the controller designed by the non-causal approach produces a faster response with less overshoot, compared to the controller designed by the causal approach, as shown in Fig. 6.

**Case 3b.** In this case, contrary to Case 3a, the actual time-delays vary within the guaranteed stability range of the causal approach, but outside the guaranteed stability range of the non-causal approach. The controller designed by the non-causal approach, however, still produces as good response as the controller designed by the causal approach, as shown in Fig. 7.

**Case 3c.** In this case, the actual time-delays vary within a range above the guaranteed stability ranges of both controllers. The controller designed by the non-causal approach can still produce a stable response as shown in Fig. 8, although some overshoot now occurs and the settling time is longer compared to the previous two cases. The controller designed by the causal approach, on the other hand, cannot stabilize the system.
In this case, different from the previous cases, the actual time-delays vary within a range which is not within the guaranteed stability range of either controller, but in a range between the two. Both controllers produce a smooth and stable response, as shown in Fig. 9. The response of the controller designed by the non-causal approach, however, is much faster.

The simulation results (not only the ones presented here, but also many others we have considered and can not present here due to space limitations—see [14] for additional cases), indicate that the non-causal approach in general produces a faster response with smaller overshoot compared to the causal approach. Interestingly, this is even true when the actual time-delays vary within the guaranteed stability range of the controller designed by the causal approach, but outside the guaranteed stability range of the controller designed by the non-causal approach (see Case 3b). Furthermore, the controllers designed by the non-causal approach have better stability robustness, in general (e.g. see Case 3c).

VI. CONCLUSION

Performance and robustness improvements obtained by utilizing the approach of [17], which uses non-causal uncertainty blocks, have been presented. Through a number of examples, it has been shown that the controllers obtained by the approach of [17] have better performance and robustness properties compared to the controllers obtained by the approach of [15,16]. The controllers obtained by the approach of [17] have higher performance levels and larger stability margins. Furthermore, the guaranteed stability range of the present controllers are centered around the nominal time-delays and the performance is optimized for the nominal, i.e., most probable time-delay. On the other hand, guaranteed stability range of the controllers designed by the approach of [15,16] may even exclude the nominal time-delay and the performance is optimized for a time-delay which is less than the nominal time-delay. As far as the controller structure and complexity or the computational burden to design the controller is concerned, there is no difference between the two approaches.

The results of the present work indicate the potential advantages of using non-causal uncertainty blocks. Such blocks may appear not only in the robust flow controller design problem, but in other robust controller design problems for systems with multiple time-delays.

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