Energy Transport

Focus on → heat transfer

Heat Transfer Mechanisms:
- Conduction
- Radiation
- Convection (mass movement of fluids)
Conduction heat transfer occurs only when there is physical contact between bodies (systems) at different temperatures by molecular motion.

Heat transfer through solid bodies is by conduction alone, whereas the heat may transfer from a solid surface to a fluid partly by conduction and partly by convection.
Fourier’s Law of Thermal Conduction

\[ q_y = -k \frac{\partial T}{\partial y} \]

- the heat flux in the y direction (W/m²)
- thermal conductivity (W/m.K)
- temperature gradient in the y-direction (K/m)

Fig. 6.1 Build-up to steady-state temperature profile for a solid slab.
The proportionality ratio, $k$, is called **thermal conductivity** of the material.
Thermal Conductivity (k)

The thermal conductivity of a substance is defined as the heat flow per unit area per unit time when the temperature decreases by one degree in unit distance.

The thermal conductivity is a material property which reflects the relative ease or difficulty of the transfer of energy through the material. It depends on the bonding and structure of the material.
Thermal Diffusivity (α)

- The thermal diffusivity is a fundamental quantity. It is analogous to momentum and mass diffusivities.

\[ \alpha = \frac{k}{\rho C_v} \]

where \( \rho \) and \( C_v \) are the density and specific heat of the material, respectively.

- In mks system, the unit of thermal diffusivity is m\(^2\).s\(^{-1}\), while in the cgs system it is usually cm\(^2\).s\(^{-1}\).
Thermal Radiation

Thermal radiation is the energy radiated from hot surfaces as electromagnetic waves. It does not require medium for its propagation. Heat transfer by radiation occur between solid surfaces, although radiation from gases is also possible. Solids radiate over a wide range of wavelengths, while some gases emit and absorb radiation on certain wavelengths only.

The energy flux emitted by an ideal radiator is proportional to the fourth power of its absolute temperature.

\[ e_b = \sigma T^4 \]

where \( e_b \) is the emissive power and \( \sigma \) is the Stefan-Boltzmann constant.
When thermal radiation strikes a body, it can be absorbed by the body, reflected from the body, or transmitted through the body. The fraction of the incident radiation which is absorbed by the body is called **absorptivity** ($\alpha$).

Other fractions of incident radiation which are reflected and transmitted are called **reflectivity** (symbol $\tau$) and **transmissivity**, $\rho$.

The sum of these fractions should be unity i.e. $\alpha + \rho + \tau = 1$. 
Convection

Convection is the heat transfer within a fluid, involving gross motion of the fluid itself.

Fluid motion may be caused by differences in density as in free convection. Density differences are a direct result of temperature differences between the fluid and the solid wall surface.

In forced convection, the fluid motion is produced by mechanical means, such as a domestic fan-heater in which a fan blows air across an electric element.
Heat Transfer Coefficient (h)

When a moving fluid at one temperature is in contact with a solid at a different temperature, heat exchanges between the solid and the fluid by conduction at a rate given by Fourier’s law. Under such cases the distribution of temperature within the fluid and the heat flux at the wall can be determined by using *heat transfer coefficient, h*,

\[ h = \frac{q_0}{T_s - T_f} = -\frac{k(dT/dy)_0}{T_s - T_f} \]

where
- \( T_s \), the surface temperature
- \( T_f \), bulk fluid temperature
- \( q_0 \), heat flux at the wall
- \( (dT/dy)_0 \), the temperature gradient in the fluid normal to the wall at the fluid-solid interface.
- \( k \), the conductivity of the fluid
Thermal Conductivity of Gases

Conduction of energy in a gas phase is primarily by transfer of translational energy from molecule to molecule as the faster moving (higher energy) molecules collide with the slower ones.

\[ k = \frac{C_v \bar{V} \lambda}{3} \]

where \( C_v \), the heat capacity per unit volume, \( \bar{V} \), the average speed, \( \lambda \), the mean free path.

\[ k = \frac{1}{d^2} \left[ \frac{\kappa_B^3 T}{\pi^3 m} \right]^{1/2} \]

where \( m \), mass of fluid molecules, \( K_B \), the Boltzmann constant, \( T \), absolute temperature, \( d \), the center to center distance of two molecules.

The thermal conductivity of gases is independent of pressure and depends only on temperature. This conclusion is valid up to about ten atmospheres (1.0133 x 10^5 Pa)
Eucken developed the following equation for the thermal conductivity of polyatomic gases at normal pressures,

$$k = \eta \left( C_p + \frac{1.25R}{M} \right)$$

where $M$, molecular weight, $C_p$, the heat capacity at constant pressure.
This figure is valid up to about ten atmospheres.

Fig. 6.2 Thermal conductivity of several gases. Data valid for up to 10^9 Pa (approx. 10 atm).
Thermal Conductivity of Gas Mixtures

The thermal conductivity of gas mixtures can be estimated within a few percent by the following equation

$$k_{mix} = \frac{\sum X_i k_i M_i^{1/3}}{\sum X_i M_i^{1/3}}$$

where $X_i$ is the mole fraction of component $i$ having molecular weight, $M_i$, and intrinsic thermal conductivity $k_i$. 
Solids transmit thermal energy by two modes:

- elastic vibrations of the lattice moving through the crystal in the form of waves

- free electrons moving through the lattice also carry energy similar to the case in gases (this is observed in metals)
Each lattice vibration (there is always a spectrum of vibrations) may be described as a traveling wave carrying energy and obeying the laws of quantum mechanics. By analogy with light theory, the waves in a crystal exhibit the characteristics of particles and are called **phonons**.

Two types of phonon-phonon interaction are observed in solids:

- **Normal or N-type**
- **Umklapp (U-process) collision**

\[
k = \eta \left( C_p + \frac{1.25R}{M} \right)
\]
Since the number of phonons increases with temperature and the wavelength of phonons $\lambda_{ph}$ is proportional to $1/T$. At room temperature and above, molar heat capacity $\hat{C}_v$ for most materials is roughly constant $\Rightarrow$ the thermal conductivity of a solid which conducts energy only by phonons, decreases with increasing temperature.

$$\lambda_{ph} = \frac{20T_m d}{\gamma^2 T}$$

where $T_m$, melting point, $T = \text{absolute temperature}$, $d$, crystal-lattice dimension, and $\gamma$, Gruneisen’s constant ($\sim 2$ for most solids at ordinary temperatures).

**THIS IS GENERALLY OBSERVED IN ELECTRICALLY INSULATING SUBSTANCES SUCH AS OXIDES (BUT NOT IN THE FORM OF POROUS, BULK MATERIALS).**
Fig. 6.3 Thermal conductivity of oxides and various insulating materials. (From A. Schack, *Industrial Heat Transfer*, Wiley, New York, NY, 1965, page 189.)
Phonons are also scattered by
- differences in isotopic masses
- chemical impurities
- dislocations
- second phases

Fig. 6.5 Thermal conductivity in the solid-solution system MgO-NiO. (From Kingery et al., *ibid.*, page 623.) Note: 1 cal s⁻¹ cm⁻¹ K⁻¹ = 418.4 W m⁻¹ K⁻¹.
In electrical conductors, in addition to phonons, conduction electrons contribute to thermal conductivity. The electronic contribution to the thermal conductivity, $k_{el}$,

$$k_{el} = \frac{\pi^2 n_e K_B T \lambda_{el}}{3 m_e \overline{V_f}}$$

where $n_e$, the number of free electrons per cm$^3$, $\lambda_{el}$ the mean free path of electron, $\overline{V_f}$, electron velocity at the Fermi surface and $m_e$, electron mass.

Wiedmann-Franz law and the Lorentz number, $L$, are utilized to determine what is the dominant mechanism for thermal conduction

$$L = \frac{k_{el}}{\sigma_e T} = 2.45 \times 10^{-8} \text{ W ohm K}^{-2}$$
Fig. 6.6 Thermal conductivities of pure, solid, and liquid metals.
Fig. 6.7 Thermal conductivities of nickel and of nickel-base alloys.
Fig. 6.8 Thermal conductivities of pure iron and iron-base alloys. (From Schack, *ibid.*)
Fig. 6.9 Thermal conductivity of high-temperature materials.
Table 6.1 Thermal conductivities of amorphous or molecular solids

<table>
<thead>
<tr>
<th>Substance</th>
<th>Temperature, K</th>
<th>$k$, W m$^{-1}$ K$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>373</td>
<td>0.76</td>
</tr>
<tr>
<td>Lead glass</td>
<td>273</td>
<td>0.87</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>373</td>
<td>1.16</td>
</tr>
<tr>
<td>Quartz glass</td>
<td>373</td>
<td>1.42</td>
</tr>
<tr>
<td>Asphalt</td>
<td>293</td>
<td>0.76</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>293</td>
<td>0.12</td>
</tr>
<tr>
<td>Polyvinyl chloride</td>
<td>293</td>
<td>0.26</td>
</tr>
</tbody>
</table>